Estimate of a Chiral-Odd Contribution to Single Transverse-Spin Asymmetry in Hadronic Pion Production

Y. Kanazawa and Yuji Koike

Department of Physics, Niigata University, Ikarashi, Niigata 950–2181, Japan

Abstract:

We present an estimate of a chiral-odd contribution to the single transverse spin asymmetry in the large-p_T pion production in the nucleon-nucleon collision. In this contribution the transversity distribution and the chiral-odd spin-independent twist-3 distribution appear. Because of the smallness of the corresponding hard cross section, this term turned out to be negligible in all kinematic regions compared with the chiral-even contribution.

PACS numbers: 12.38.-t, 12.38.Bx, 13.85.Ni, 13.88.+e
[Keywords: Single transverse spin asymmetry, Twist three, Chiral-odd]

Single transverse spin asymmetries for the pion production with large transverse momentum in pp collision

$$N'(P', \vec{S}_\perp) + N(P) \to \pi(\ell) + X, \tag{1}$$

have been receiving great attention [1]-[7], in particular, after the FNAL E704 data showed a large asymmetry [8]. Ongoing experiment at RHIC is expected to provide more data on the asymmetry. The process probes particular quark-gluon correlations (higher twist effect) [1, 6, 7, 9] in the nucleon not included in the twist-2 parton densities. According to the generalized QCD factorization theorem [10], the polarized cross section for this process consists of three types of twist-3 contributions:

$$(\mathrm{A}) \qquad G_a(x_1', x_2') \otimes q_b(x) \otimes D_{c\to\pi}(z) \otimes \hat{\sigma}_{ab\to c}, \tag{2}$$

$$(\mathrm{B}) \qquad \delta q_a(x') \otimes E_b(x_1, x_2) \otimes D_{c\to\pi}(z) \otimes \hat{\sigma}'_{ab\to c}, \tag{3}$$

$$(\mathrm{C}) \qquad \delta q_a(x') \otimes q_b(x) \otimes D^{(3)}_{c\to\pi}(z_1, z_2) \otimes \hat{\sigma}''_{ab\to c}. \tag{4}$$

Here the functions $G_a(x_1', x_2')$, $E_b(x_1, x_2)$ and $D^{(3)}_{c\to\pi}(z_1, z_2)$ are the twist-3 quantities representing, respectively, the transversely polarized distribution, the unpolarized distribution, and the fragmentation function for the pion, and a, b and c stand for the parton's species. Other functions are twist-2; $q_b(x)$ the unpolarized distribution (quark or gluon) and $\delta q_a(x)$ the transversity distribution, etc. The symbol $\otimes$ denotes convolution. $\hat{\sigma}_{ab\to c}$ *etc* represents the partonic cross section for the process $a + b \to c + \mathit{anything}$ which yields large transverse momentum of the parton c. Note that δq_a, E_b and $D^{(3)}_{c\to\pi}$ are chiral-odd, and (B) and (C) contain two chiral-odd functions.

Qiu and Sterman [6] presented a first systematic QCD analysis on the chiral-even contribution (A). They showed that at large x_F, i.e., pion production in the forward direction with respect to the polarized nucleon beam which mainly probes large x' and small x region, the cross section is dominated by the particular terms in (A) which contain the derivatives of the *valence* twist-3 distribution $G_{Fa}(x', x')$. The main reason for this observation is the relation $|\frac{\partial}{\partial x'} G_{Fa}(x', x')| \gg G_{Fa}(x', x')$ owing to the behavior of $G_{Fa}(x', x') \sim (1 - x')^\beta$ $(\beta > 0)$ at $x' \to 1$. Keeping only those terms (*valence quark-soft gluon approximation*) together with a moderate model assumption for G_{Fa}, they reproduced the rising behavior of the E704 data toward $x_F \to 1$ reasonablly well. In a recent paper[7], we extended the analysis to one of the chiral-odd contribution ((B) term) and presented a cross section formula in this valence qaurk-soft gluon approximation. We also discussed a possibility that this term could be a large source of the asymmetry at large *negative* x_F in parallel with the argument for the chiral-even contribution. In this report, we shall present an actual estimate of the chiral-odd contribution (B) in comparison with the chiral-even one (A). We will see that unlike the previous expectation the chiral-odd one derived in [7] is negligible in all kinematic range due to the smallness of the hard cross section, even though the derivative brings an enhancement to $E_b(x, x)$.

The polarized cross section for (1) is a function of three independent variables, $S = (P+P')^2 \simeq 2P\cdot P'$, $x_F = 2\ell_\parallel/\sqrt{S}$ ($= (T-U)/S$ below), and $x_T = 2\ell_T/\sqrt{S}$. $T = (P'-\ell)^2 \simeq -2P'\cdot\ell$ and $U = (P-\ell)^2 \simeq -2P\cdot\ell$ are given in terms of these three variables by $T = -S\left[\sqrt{x_F^2+x_T^2} - x_F\right]/2$ and $U = -S\left[\sqrt{x_F^2+x_T^2}+x_F\right]/2$. In this convention, production of the pion in the forward (backward) hemisphere in the direction of the polarized nucleon corresponds to $x_F > 0$ ($x_F < 0$). Since $-1 < x_F < 1$, $0 < x_T < 1$ and $\sqrt{x_F^2+x_T^2} < 1$, $x_F \to 1$ corresponds to the region with $-U \sim S$ and $T \sim 0$, and $x_F \to -1$ corresponds to the region with $-T \sim S$ and $U \sim 0$.

In the valence quark-soft gluon approximation, the cross section formula for (A) and (B) terms read, respectively [6, 7]

$$E_\pi \frac{d^3\Delta\sigma^A(S_\perp)}{d\ell^3} = \frac{\pi M \alpha_s^2}{S}\sum_{a,c}\int_{z_{min}}^1 \frac{dz}{z^3} D_{c\to\pi}(z)\int_{x'_{min}}^1 \frac{dx'}{x'}\frac{1}{x'S+U/z}\int_0^1 \frac{dx}{x}\delta\left(x+\frac{x'T/z}{x'S+U/z}\right)$$
$$\times\, \epsilon_{\ell S_\perp pn}\left(\frac{1}{-\hat{u}}\right)\left[-x'\frac{\partial}{\partial x'}G_{Fa}(x',x')\right]\left[G(x)\Delta\widehat{\sigma}_{ag\to c} + \sum_b q_b(x)\Delta\widehat{\sigma}_{ab\to c}\right], \qquad (5)$$

$$E_\pi \frac{d^3\Delta\sigma^B(S_\perp)}{d\ell^3} = \frac{\pi M \alpha_s^2}{S}\sum_{a,b,c}\int_{z_{min}}^1 \frac{dz}{z^3} D_{c\to\pi}(z)\int_{x_{min}}^1 \frac{dx}{x}\frac{1}{xS+T/z}\int_0^1 \frac{dx'}{x'}\delta\left(x'+\frac{xU/z}{xS+T/z}\right)$$
$$\times\, \epsilon_{\ell S_\perp pn}\left(\frac{1}{-\hat{t}}\right)\left[-x\frac{\partial}{\partial x}E_{Fb}(x,x)\right]\delta q_a(x')\delta\widehat{\sigma}_{ab\to c}, \qquad (6)$$

where p and n are the two light-like vectors defined from the momentum of the unpolarized nucleon as $P = p + M^2 n/2$, $p\cdot n = 1$ and $\epsilon_{\ell S_\perp pn} = \epsilon_{\mu\nu\lambda\sigma}\ell^\mu S_\perp^\nu p^\lambda n^\sigma \sim \sin\phi$ with ϕ the azimuthal angle between the nucleon spin vector and the scattering plane. The invariants in the parton level are defined as

$$\hat{s} = (p_a+p_b)^2 \simeq (x'P'+xP)^2 \simeq xx'S,$$
$$\hat{t} = (p_a-p_c)^2 \simeq (x'P'-\ell/z)^2 \simeq x'T/z,$$
$$\hat{u} = (p_b-p_c)^2 \simeq (xP-\ell/z)^2 \simeq xU/z, \qquad (7)$$

and the lower limits for the integration variables are

$$z_{min} = \frac{-(T+U)}{S} = \sqrt{x_F^2+x_T^2},$$
$$x_{min} = \frac{-T/z}{S+U/z}, \qquad x'_{min} = \frac{-U/z}{S+T/z}. \qquad (8)$$

Equation (5) is derived in [6] with the unpolarized gluon distribution $G(x)$ and the partonic cross section $\Delta\widehat{\sigma}_{ag\to c}$ and $\Delta\widehat{\sigma}_{ab\to c}$. $G_F(x',x')$ is the soft gluon component of the twist-3

transversely polarized distribution:

$$G_{Fa}(x,x) = \frac{1}{M}\epsilon_{S_\perp \sigma pn}\int \frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS|\bar{\psi}^a(0)\not{n}\left\{\int \frac{d\mu}{2\pi} gF^{\sigma\beta}(\mu n)n_\beta\right\}\psi^a(\lambda n)|PS\rangle, \qquad (9)$$

where $\epsilon_{S_\perp \sigma pn} \equiv \epsilon_{\mu\sigma\nu\lambda}S_\perp^\mu p^\nu n^\lambda$. Equation (6) is derived in [7] with the partonic cross section $\delta\hat{\sigma}_{ab\to c}$. $E_F(x,x)$ is the soft gluon component of the unpolarized twist-3 distrbution defined as

$$E_{Fa}(x,x) = \frac{-i}{2M}\int \frac{d\lambda}{2\pi}e^{i\lambda x}\langle P|\bar{\psi}^a(0)\not{n}\gamma_{\perp\sigma}\left\{\int \frac{d\mu}{2\pi} gF^{\sigma\beta}(\mu n)n_\beta\right\}\psi^a(\lambda n)|P\rangle. \qquad (10)$$

The summation for the flavor indices of $G_{Fa}(x',x')$ and $E_{Fa}(x,x)$ is to be over u- and d-valence quarks, while that for the twist-2 distributions is over u, d, $\bar{u}$, $\bar{d}$, s, $\bar{s}$. $\Delta\hat{\sigma}_{ab\to c}$ and $\Delta\hat{\sigma}_{ag\to c}$ can be obtained from the $2\to 2$ diagrams shown in Figs. 1 and 2, respectively. $\delta\hat{\sigma}_{ab\to c}$ is also obtained from diagrams in Fig. 1 but with different spin projection from the chiral even case. Because of the chiral-odd nature of $\delta q(x')$ and $E_F(x,x)$, they have to form a closed Fermion loop together, and accordingly the contribution from the diagrams shown in Figs. 1(a) and (b) vanishes. Gluon contribution shown in Fig. 2 is also absent in (6).

At large $|x_F|$, one can easily guess the relative magnitude of each diagram shown in Fig.1. The gluon propagators in Figs. 1(a), (b), (c) and (d) give rise to the factors $1/\hat{t}^2$, $1/\hat{u}^2$, $1/\hat{s}^2$ and $1/\hat{t}\hat{u}$. At $x_F\to 1$, $\hat{t}$ becomes very small and the hard cross section contribution from Fig.1(a) becomes much larger than the others. At $x_F\to -1$ the role of $\hat{t}$ and $\hat{u}$ is interchanged. By the same reason, the gluon contribution (Fig.2) also becomes very large. Accordingly the hard cross sections for (6) at $x_F\to -1$ is much smaller than those for (5). For completeness we list the partonic cross sections here. They read

$$\begin{aligned}
\Delta\hat{\sigma}_{ab\to c} &= \frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)\left[\frac{1}{4}+\frac{1}{8}\left(1+\frac{\hat{u}}{\hat{t}}\right)\right]\delta_{ac} + \frac{4}{9}\left(\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)\left[\frac{1}{4}-\frac{7}{8}\left(1+\frac{\hat{u}}{\hat{t}}\right)\right]\delta_{bc} \\
&\quad + \frac{-8}{27}\left(\frac{\hat{s}^2}{\hat{t}\hat{u}}\right)\left[\frac{10}{8}+\frac{1}{8}\left(1+\frac{\hat{u}}{\hat{t}}\right)\right]\delta_{ac}\delta_{bc}, \\
\Delta\hat{\sigma}_{a\bar{b}\to c} &= \frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)\left[\frac{7}{8}+\frac{1}{8}\left(1+\frac{\hat{u}}{\hat{t}}\right)\right]\delta_{ac} + \frac{-4}{9}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)\left[\frac{1}{8}+\frac{7}{8}\left(1+\frac{\hat{u}}{\hat{t}}\right)\right]\delta_{ab}, \\
\Delta\hat{\sigma}_{a\bar{b}\to \bar{c}} &= \frac{4}{9}\left(\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)\left[\frac{7}{8}-\frac{1}{4}\left(1+\frac{\hat{u}}{\hat{t}}\right)\right]\delta_{bc} + \frac{-4}{9}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)\left[\frac{1}{8}+\frac{1}{4}\left(1+\frac{\hat{u}}{\hat{t}}\right)\right]\delta_{ab}, \qquad (11)
\end{aligned}$$

for the chiral-even one[1] and

$$\delta\hat{\sigma}_{ab\to c} = \left[\frac{10}{27}+\frac{1}{27}\left(1+\frac{\hat{t}}{\hat{u}}\right)\right]\delta_{ab}\delta_{bc},$$

[1]The cross sections listed in eq. (76b) of Ref.[6] is not correct, since some of the color factors in table II of [6] is wrong. Numerically, however, the difference between (11) and eq.(76b) of [6] is tiny. We thank Jianwei Qiu for communication to clarify this point. $\Delta\hat{\sigma}_{ag\to c}$ is given in eq.(76a) of [6].

$$\delta\widehat{\sigma}_{\bar{a}b\to c} = -\frac{\hat{t}\hat{u}}{\hat{s}^2}\left[\frac{1}{9}+\frac{7}{9}\left(1+\frac{\hat{t}}{\hat{u}}\right)\right]\delta_{ab},$$

$$\delta\widehat{\sigma}_{\bar{a}b\to\bar{c}} = -\frac{\hat{t}\hat{u}}{\hat{s}^2}\left[\frac{1}{9}+\frac{2}{9}\left(1+\frac{\hat{t}}{\hat{u}}\right)\right]\delta_{ab}, \tag{12}$$

for the chiral-odd one [7] [2]. Even though the hard cross section for the chiral-odd term (B) is small, the distribution function receives a large enhancement by the derivative of $E_F(x,x)$ in (6). Employing a simple model assumption introduced in [7], we shall present a numerical estimate of the asymmetry.

Our model assumption for $G_F(x,x)$ and $E_F(x,x)$ is based on the comparison of their explicit form (9) and (10) with the twist-2 distributions

$$q_a(x) = \frac{1}{2}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P|\bar{\psi}^a(0)\not{n}\psi^a(\lambda n)|P\rangle, \tag{13}$$

$$\delta q_a(x) = \frac{i}{2}\epsilon_{S_\perp\sigma pn}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS|\bar{\psi}^a(0)\not{n}\gamma_\perp^\sigma\psi^a(\lambda n)|PS\rangle. \tag{14}$$

We make an ansatz

$$G_{Fa}(x,x) = K_a q_a(x), \tag{15}$$

$$E_{Fa}(x,x) = K_a \delta q_a(x), \tag{16}$$

with a flavor-dependent parameter K_a which represents the effect of the gluon field with zero momentum in $G_F(x,x)$ and $E_F(x,x)$. We note that even though $E_F(x,x)$ is an unpolarized distribution, the quarks in $E_F(x,x)$ is "transversely polarized". One may interpret that these polarized quarks in unpolarized nucleon would be a source of the asymmetry in the forward direction of the unpolarized nucleon ($x_F\to -1$) in (B).

Following [6], we determine $K_{u,d}$ to fit the E704 data at $x_F>0$. Since we are only interested in the estimate of order of magnitude, we ignore the scale dependence of each distribution and fragmentation function. For the unpolarized distribution $q_a(x)$, we use the GRV LO distribution at the input scale $\mu^2 = 0.23$ GeV^2 [11]. For the fragmentation function of the pion, we use the one given in [12] at the input scale $\mu^2 = 2.0$ GeV^2. The result for the single transverse spin asymmetry A_T is shown in Fig.3 with $K_u = -K_d = 0.07$. The data is reasonablly well reproduced. Using these values for $K_{u,d}$, we calculated the asymmetry A_T at $x_F<0$ region. For the transversity distribution, we use the GRSV helicity distribution $\Delta q_a(x)$ (LO, valence scenario)[13] assuming $\delta q_a(x) = \Delta q_a(x)$ at the input scale. The result is shown in Fig. 4. For comparison, we also plotted the chiral-even contribution (5) for π^+, although this term is not a dominant contribution among all chiral-even ones at $x_F<0$. For all $\pi^{\pm,0}$, it turns out that (6) gives rise to negligible asymmetry, even smaller than a

[2] $\delta_{\bar{a}b}$ in the second and third equations in eq.(17) of [7] should be changed into δ_{ab} as corrected above.

part of the chiral-even contribution in the negative x_F region. This is because the hard cross section (12) is much smaller than (11).

In the region of large x_T with $x_F \sim 0$, the polarized cross section for (1) probes the region of $x \sim x' \sim 1$. Accordingly the valence quark-soft gluon approximation again becomes valid. This kinematic region corresponds to $T \sim U \sim -S/2$ and hence the magnitude of each contribution in Fig.2 becomes comparable. We therefore plotted (5) and (6) in Fig.5 separately. One sees that even in this region the chiral-even contribution is much larger than the chiral-odd one. Experimentally, this region is almost impossible to achieve.

Another chiral-odd contribution (C) is yet to be analyzed for a complete description of the single transverse asymmetry (1). The calculation of the hard cross section for this term is again reduced to the calculation of $2 \to 2$ scattering diagrams shown in Figs. 1 and 2. For this case the diagrams shown in Figs. 1 (a) and (b) and Fig. 2 also contribute, and hence may cause a large effect to the asymmetry. The analysis of this term will be reported elsewhere.

A different approach to the single spin asymmetry introduces the so-called T-odd distribution or fragmentation functions with the intrinsic transverse momentum instead of twist-3 distributions introduced here [2, 3, 14, 15]. Corresponding to (A), (B) and (C), this approach starts from the factoriztion assumption for the three types of contributions to the asymmetry: (i) $f_{1T}^{\perp}(x', \boldsymbol{p}'_{\perp}) \otimes q(x) \otimes D(z) \otimes \hat{\sigma}$, (ii) $\delta q(x') \otimes h_1^{\perp}(x, \boldsymbol{p}_{\perp}) \otimes D(z) \otimes \hat{\sigma}'$, (iii) $\delta q(x') \otimes q(x) \otimes H_1^{\perp}(z, \boldsymbol{k}_{\perp}) \otimes \hat{\sigma}''$, where $f_{1T}^{\perp}$ represents distribution of an unpolarized quark with nonzero transverse momentum inside a transversely polarized nucleon [2], $h_1^{\perp}$ represents distribution of a transversely polarized quark with nonzero tranverse monetum inside the unpolarized nucleon [14], and $H_1^{\perp}$ represents a fragmentation function for a transversely polarized quark fragmenting into a pion with the transvere momentum[3]. Anselmino *et al* fitted the E704 data for the asymmetry assuming the above (i) or (iii) are the sole origin of the asymmetry. It is clear from the present study that the second contribution (ii) is tiny due to the smallness of the partonic cross section also in this approach. For the lowest order Drell-Yan single spin asymmetry, it has been shown that the T-odd function $f_{1T}^{\perp}$ and $h_1^{\perp}$ effectively play the role of the soft gluon pole contribution considered here [15]. It is interesting to explore the connection between the present approach and those in [2, 3, 14, 4].

To summerize, we have estimated the magnitude of one of the chiral-odd contribution to the single transverse spin asymmetry of the hadronic pion production. Due to the smallness of the partonic hard cross section, it turns out that this term is negligible in all kinematic region.

References

[1] A.V. Efremov and O.V. Teryaev, Sov. J. Nucl. Phys. **36** (1982) 140; Phys. Lett. **B150** (1985) 383.

[2] D. Sivers, Phys. Rev. **D41** (1990) 83; Phys. Rev. **D43** (1991) 261.

[3] J.C. Collins, Nucl. Phys. **B396** (1993) 161.

[4] M. Anselmino, M. Boglione, F. Murgia, Phys. Lett. **B362** (1995) 164; Phys. Rev. **D60** (1999) 054027;

[5] X. Artru, J. Czyzewski and H. Yabuki, Z. Phys. **C73** (1997) 527.

[6] J. Qiu and G. Sterman, Phys. Rev. **D59** (1999) 014004.

[7] Y. Kanazawa and Y. Koike, Phys. Lett. **B478** (2000) 121.

[8] D.L. Adams et al., Phys. Lett. **B261** (1991) 201; **B264** (1991) 462; A. Bravar et al., Phys. Rev. Lett. **77** (1996) 2626.

[9] J. Qiu and G. Sterman, Phys. Rev. Lett. **67** (1991) 2264; Nucl. Phys. **B378** (1992) 52.

[10] J.C. Collins, D.E. Soper and G. Sterman, in *Perturbative Quantum Chromodynamics*, ed. A.H. Mueller (World Scientific, Singapore, 1989) p.1.; J. Qiu and G. Sterman, Nucl. Phys. **B353** (1991) 137.

[11] M. Glück, E. Reya and A. Vogt, Z. Phys. **C67** (1995) 433.

[12] J. Binewise, B.A. Kniehl and G. Kramer, Z. Phys. **C65** (1995) 471.

[13] M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. **D53** (1996) 4775.

[14] D. Boer, Phys. Rev.**D60** (1999) 014012.

[15] D. Boer, P.J. Mulders and O.V. Teryaev, Phys. Rev. **D57** (1998) 3057.

Figure Captions

Fig. 1 Quark-quark $2 \to 2$ scattering diagrams contributing to the hard cross section.

Fig. 2 Quark-gluon $2 \to 2$ scattering diagrams contributing to the hard cross section.

Fig. 3 The E704 data for the single transverse spin asymmetry A_T in the inclusive pion production in the $x_F > 0$ region at $\sqrt{s} = 20$ GeV [8]. The transverse momentum of the pion is $\ell_T = 1.5$ GeV. The curves show (5) calculated with the method described in the text.

Fig. 4 Single transverse spin asymmetry A_T in the inclusive pion production at $x_F < 0$. The transverse momentum of the pion is calculated at $\ell_T = 1.5$ GeV. The curves obtained from (6) can not be distinguished from zero for all $\pi^{\pm,0}$. The chiral-even contribution (5) is also shown by dotted line for π^+ for comparison.

Fig. 5 Single transverse spin asymmetry A_T in the inclusive pion production as a fuction of x_T at $x_F = 0$. Solid and dashed lines, respectively, stand for the contribution from (5) and (6).

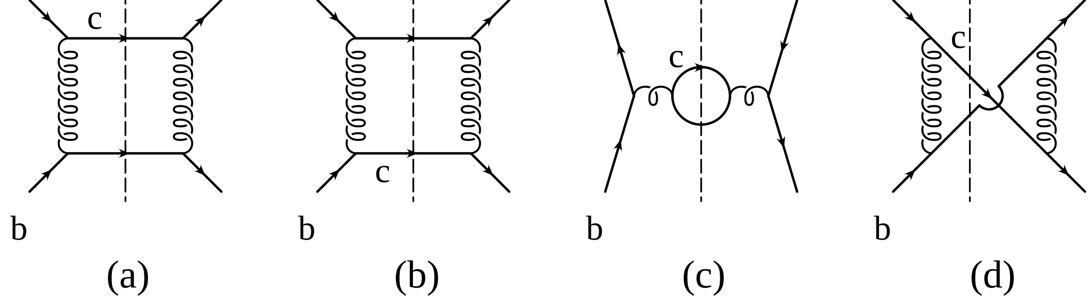


Fig.1

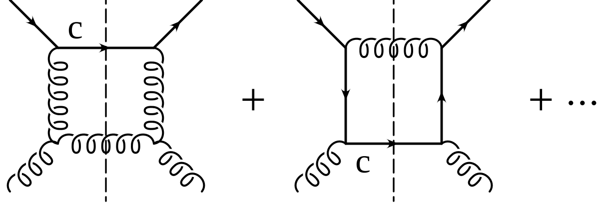


Fig.2

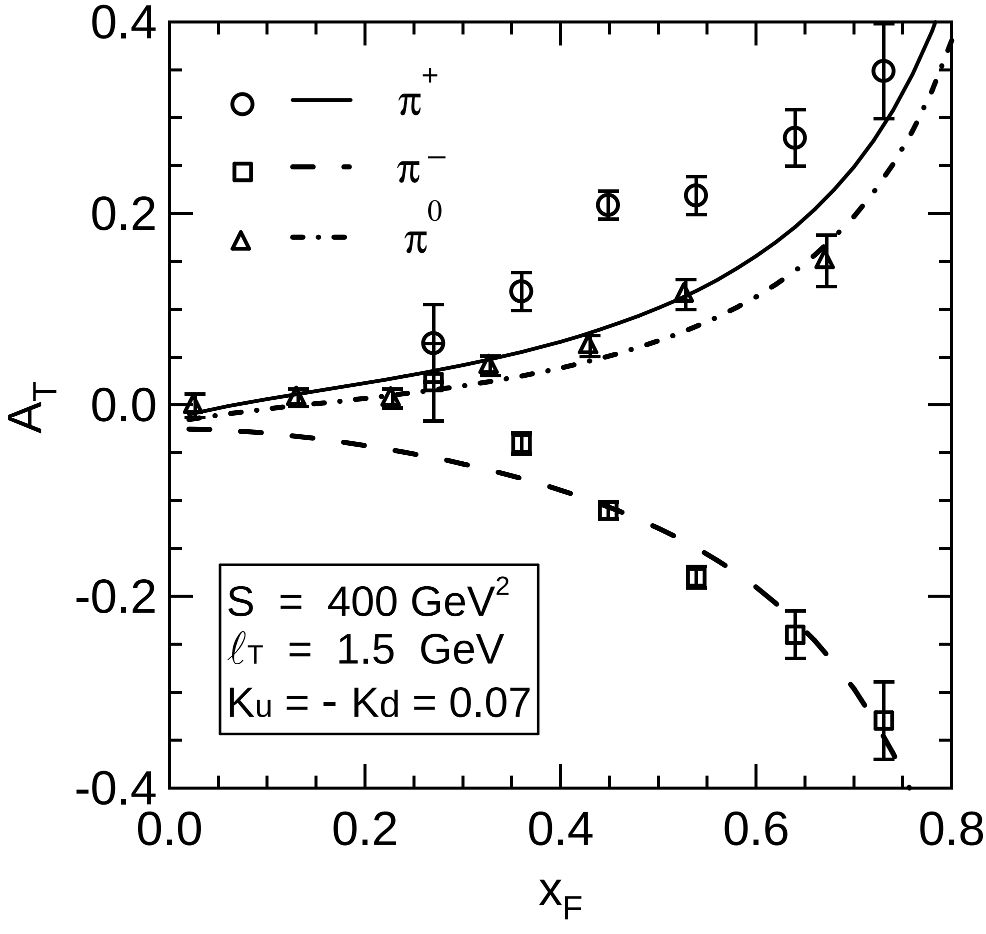
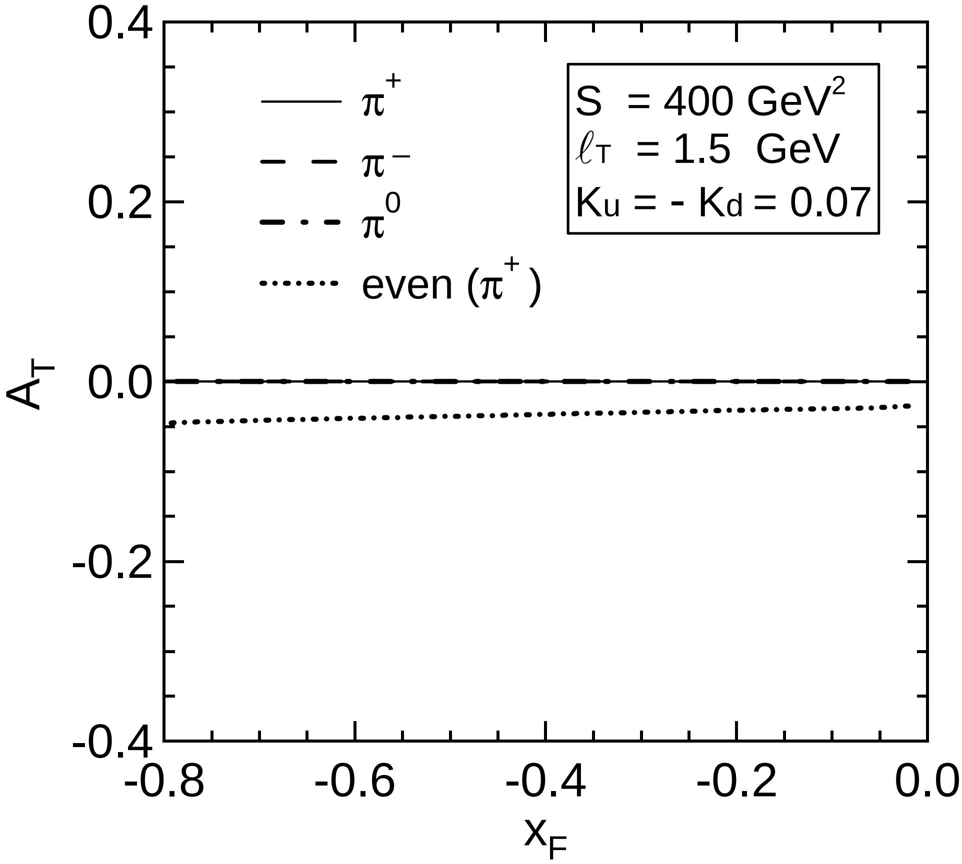

0.4
0.2
0.0
-0.2
-0.4
0.0
0.2
0.4
0.6
0.8
A_T
x_F
π^+
π^-
π^0
S = 400 GeV^2
ℓ_T = 1.5 GeV
Ku = - Kd = 0.07

Fig.3



0.4
0.2
0.0
-0.2
-0.4
-0.8
-0.6
-0.4
-0.2
0.0
A_T
x_F
π^+
π^-
π^0
even (π^+)
S = 400 GeV^2
ℓ_T = 1.5 GeV
Ku = - Kd = 0.07

Fig.4

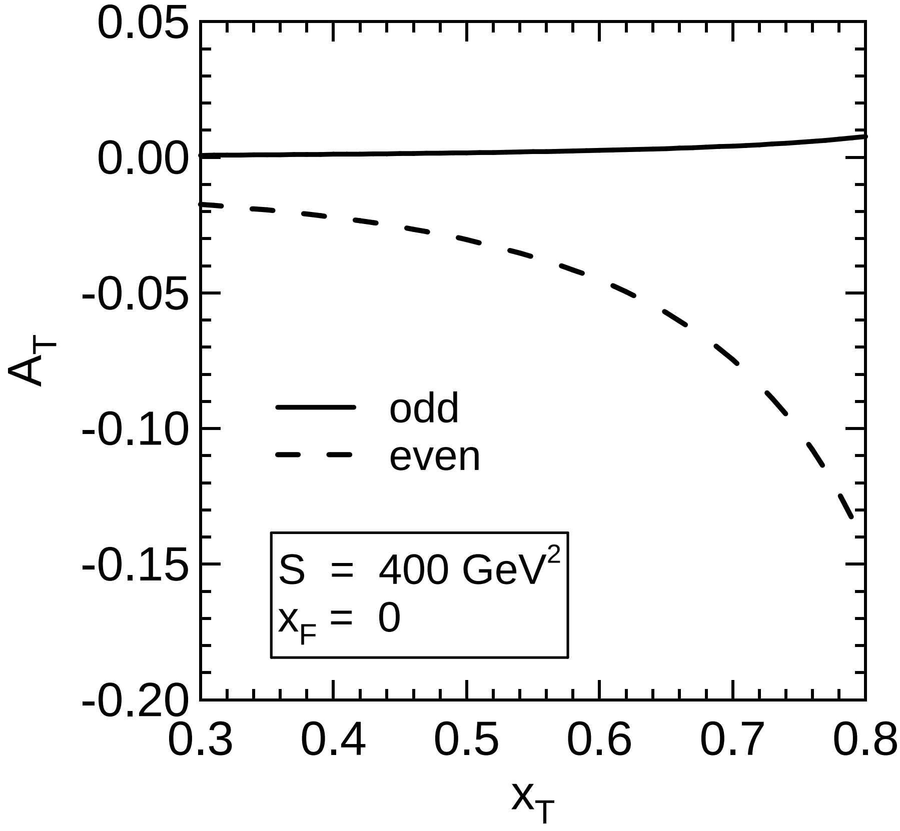

0.05
0.00
-0.05
-0.10
-0.15
-0.20
0.3
0.4
0.5
0.6
0.7
0.8
A_T
x_T
odd
even
S = 400 GeV^2
x_F = 0

Fig.5